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Generation of Bright Two-Color Continuous Variable Entanglement

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We present the first measurement of squeezed-state entanglement between the twin beams produced in an optical parametric oscillator operating above threshold. In addition to the usual squeezing in the intensity difference between the twin beams, we have measured squeezing in the sum of phase quadratures. Our scheme enables us to measure such phase anticorrelations between fields of different frequencies. In the present measurements, wavelengths differ by ≈ 1 nm. Entanglement is demonstrated according to the Duan *et al.* criterion [Phys. Rev. Lett. **84**, 2722 (2000)] $\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.41(2) < 2$. This experiment opens the way for new potential applications such as the transfer of quantum information between different parts of the electromagnetic spectrum.

The field of quantum information has recently attracted great interest, owing to potential applications in information storage, communication, and computing [1]. Entanglement is viewed as a key resource for these applications, especially for quantum communication. A variety of physical systems presenting entanglement have been investigated both theoretically and experimentally. The vast majority of experiments concentrate on discrete variable systems, such as trapped ions [2], few photon electromagnetic fields in cavity QED [3], spontaneous parametric down-conversion [4], and nuclear magnetic resonance [5]. On the other hand, in recent experiments, continuous variable systems have been studied, including light beams [6–9] and atomic samples [10]. The number of such experiments, however, is still relatively small in comparison with discrete variable systems.

The first experimental demonstration of continuous variable entanglement used a continuous-wave (CW) optical parametric oscillator (OPO) operating below threshold [6]. The two squeezed vacuum outputs were shown to possess Einstein Podolsky Rosen (EPR) type correlations. Most recent experiments use a nonlinear interaction to produce squeezed fields, which are then combined in a beam splitter to generate entanglement [11]. Conversely, a beam splitter transformation can also be used to generate a squeezed beam from an entangled input [12]. In these experiments, it is mandatory to have fields of the same frequency.

In this Letter, we present the first (to our knowledge) measurement of continuous variable entanglement between bright fields of truly different frequencies. Even before the first experiment [6], it was predicted that the above-threshold OPO should produce entangled twin beams [13]. So far, this prediction had not yet been verified, owing to the difficulty of measuring phase properties of the twin beams. We have succeeded in measuring quantum anticorrelations between the phase quadratures of nondegenerate twin beams.

Bipartite continuous variable entanglement [14] can be tested according to a criterion established by Duan *et al.*

[15] and also by Simon [16]. The criterion is based on the total variance of EPR-type operators. For operators $\hat{x}_i$ and $\hat{p}_i$ that obey position-momentum commutation relations, they consider the variances of combinations such as $\hat{x}_1 + \hat{x}_2$ and $\hat{p}_1 - \hat{p}_2$. The quadratures of electromagnetic fields satisfy such commutation relations. We focus here on the so-called amplitude $\hat{p}$ and phase $\hat{q}$ field quadratures, defined by

$$\hat{a} = \frac{e^{i\phi}}{2}(\hat{p} + i\hat{q}), \qquad (1)$$

where $\hat{a}$ is the field annihilation operator, ϕ is an arbitrary phase, and, for a macroscopic field with a well-defined mean amplitude, $\langle \hat{q} \rangle = 0$. In terms of these operators, inseparability (entanglement) is demonstrated by a violation of the inequality:

$$\Delta^2 \left(\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{2}} \right) + \Delta^2 \left(\frac{\hat{q}_1 + \hat{q}_2}{\sqrt{2}} \right) \geq 2, \qquad (2)$$

where the standard quantum level (SQL) is normalized to 1 for each combination of quadratures. In order to simplify notation, we refer to $(\hat{p}_1 \pm \hat{p}_2)/\sqrt{2}$ as $\hat{p}_\pm$ and to $(\hat{q}_1 \pm \hat{q}_2)/\sqrt{2}$ as $\hat{q}_\pm$. If both $\hat{p}_-$ and $\hat{q}_+$ are squeezed [$\Delta^2 \hat{p}_- < 1$ and $\Delta^2 \hat{q}_+ < 1$], inequality (2) is violated and we have squeezed-state entanglement.

It is easy to understand why entanglement is expected in the CW OPO operating above threshold. The OPO consists of a $\chi^{(2)}$ nonlinear crystal inside a resonant cavity, in which parametric down-conversion takes place. The cavity feeds back the down-converted fields, leading to stimulated parametric gain and hence to an oscillation threshold. Since the primary down-conversion process involves creating pairs of photons by annihilation of pump photons, one naturally expects strong correlations between the intensities of the twin beams: positive intensity fluctuations of one beam correspond to positive intensity fluctuations of the other beam. These correlations, however, are frequency dependent. For power spectrum analysis frequencies larger than

the OPO cavity bandwidth, correlations tend to disappear since, for times shorter than the cavity lifetime, a photon can exit the cavity while its "twin" still remains inside. Squeezing in the intensity difference was already observed back in 1987 [17]. On the other hand, energy conservation ($\omega_0 = \omega_1 + \omega_2$, where indices 0, 1, and 2 refer to pump, signal, and idler beams, respectively) and phase matching imply strong anticorrelations between phase fluctuations: positive phase fluctuations of one beam correspond to negative phase fluctuations of its "twin." This is exactly the situation discussed following (2): entanglement occurs if these (anti)correlations lead to fluctuations below the SQL. However, squeezing in the phase sum had not been measured to date.

The first prediction of entanglement in the above-threshold OPO was made by Reid and Drummond [13]. A recent detailed prediction, taking into account the effects of pump noise and cavity detunings for the three fields, for a triply resonant OPO, was presented in [18]. The basic difficulty in measuring (phase) quadrature fluctuations, in contrast with intensity fluctuations, is that the standard technique, homodyne detection, calls for a local oscillator having a well-defined phase relationship with the field to be measured. In the OPO, this is difficult to implement, since the frequencies of the twin beams are usually different, depending on the oscillating modes, and vary from one realization to the next. Hence, two local oscillators would be required: one for each beam. It would also be necessary to phase lock these fields to the twin beams.

One way to overcome this difficulty is to force the OPO to oscillate in a strictly frequency-degenerate situation. This is technically challenging and has been done by two groups, using different approaches [19,20]. Our strategy is to perform self-homodyne measurements, without the use of local oscillators, by a frequency-dependent reflection of each beam [21]. If one considers the field as a mean value at a carrier frequency with noise sidebands at some analysis frequency, a frequency-dependent reflectivity entails different phase shifts for the carrier and sidebands. Consequently, different quadrature fluctuations can be projected onto amplitude fluctuations (with respect to the phase of the mean field). A detailed description for a single field reflected off an optical cavity was given by Galatola *et al.* [22]. For an imperfect cavity, in which vacuum leaks from the outside through the mirrors, the reflected beam amplitude noise power spectrum $S_R(\Omega)$ can be written as [18]

$$S_R(\Omega) = |g_p|^2 S_p(\Omega) + |g_q|^2 S_q(\Omega) + |g_{vp}|^2 + |g_{vq}|^2,$$

(3)

where $S_p(\Omega)$ and $S_q(\Omega)$ are the incident beam amplitude and phase noise, respectively, and g_p, g_q, g_{vp}, and g_{vq} are coefficients that depend on cavity reflection and transmission coefficients through the relations:

$$g_p = \frac{1}{2}\left[\frac{r^*(\Delta)}{|r(\Delta)|} r(\Delta + \Omega) + \frac{r(\Delta)}{|r(\Delta)|} r^*(\Delta - \Omega)\right],$$

$$g_q = \frac{1}{2}\left[\frac{r^*(\Delta)}{|r(\Delta)|} r(\Delta + \Omega) - \frac{r(\Delta)}{|r(\Delta)|} r^*(\Delta - \Omega)\right],$$

$$g_{vp} = \frac{1}{2}\left[\frac{t^*(\Delta)}{|t(\Delta)|} t(\Delta + \Omega) + \frac{t(\Delta)}{|t(\Delta)|} t^*(\Delta - \Omega)\right],$$

$$g_{vq} = \frac{1}{2}\left[\frac{t^*(\Delta)}{|t(\Delta)|} t(\Delta + \Omega) - \frac{t(\Delta)}{|t(\Delta)|} t^*(\Delta - \Omega)\right].$$

(4)

Amplitude reflection $r(\Delta)$ and transmission $t(\Delta)$ coefficients can be simply written as

$$r(\Delta) = \frac{r_1 - r_2 \exp(i\Delta/\delta\nu_{ac})}{1 - r_1 r_2 \exp(i\Delta/\delta\nu_{ac})},$$

$$t(\Delta) = \frac{t_1 t_2 \exp(i\Delta/\delta\nu_{ac})}{1 - r_1 r_2 \exp(i\Delta/\delta\nu_{ac})},$$

(5)

where Δ is the detuning between the incident field central frequency and the cavity resonance frequency, $\delta\nu_{ac}$ is the cavity bandwidth (FWHM), and Ω is the analysis frequency. Cavity input mirror amplitude reflection and transmission coefficients are denoted by r_1 and t_1, while t_2 is defined so that all internal losses A obey the relation $t_2^2 = A = 1 - r_2^2$.

For analysis frequencies larger than $\sqrt{2}\delta\nu_{ac}$, it is possible to completely convert incident phase fluctuations into amplitude fluctuations of the reflected beam. As a matter of fact, if this condition is satisfied, then, for $\Delta = \pm\delta\nu_{ac}/2$, $|g_q|^2 \approx 1$ and incident phase fluctuations are projected onto amplitude fluctuations of the reflected beam. For $\Delta = 0$ and for $|\Delta| \gg \delta\nu_{ac}$, then $|g_p|^2 \approx 1$ and we recover amplitude fluctuations. We use one analysis cavity for each beam and scan their frequencies synchronously. In this way, we are always measuring the same quadrature for each field, with respect to its mean value, regardless of the frequency difference between the fields.

Our experiment is performed with a triply resonant CW OPO operating above threshold [23]. The pump laser is an ultrastable diode-pumped frequency-doubled Nd:YAG source (Innolight Diabolo) at 532 nm, with a second output beam at 1064 nm, which we use for alignment purposes. The nonlinear crystal is a 12 mm long type-II High Gray Tracking Resistant Potassium Titanyl Phosphate (KTP) from Raicol. Crystal temperature is kept near 24 °C, with a stability of the order of 10 mK, by means of a peltier element. The cavity is a quasiconfocal linear Fabry-Perot cavity, with input mirror reflectivities equal to 89% at 532 nm and greater than 99.8% at 1064 nm. Output mirror reflectivities are greater than 99.8% at 532 nm and 95% at 1064 nm. The typical threshold power is 60 mW, and the OPO cavity bandwidth is $\delta\nu_{OPO} = 53(3)$ MHz. Noise in the difference of signal and idler intensities has a stable value $\Delta^2\hat{p}_- = 0.59(1)$ registered by the photodetectors, or $-2.30(5)$ dB. According to [18], $\Delta^2\hat{q}_+$ is significantly

affected by pump excess phase noise. Our laser presented excess noise up to 25 MHz, which we had to filter by transmission through a ring cavity, with a bandwidth $\delta\nu_f = 2.3(1)$ MHz. In this way, the pump beam was shot-noise limited for frequencies above 15 MHz.

The experimental setup is sketched in Fig. 1. The pump beam at 532 nm is sent through the filter cavity and then mode matched to the OPO cavity. The orthogonally polarized twin beams produced, with powers of the order of a few milliwatts each, are separated by a polarizing beam splitter (PBS) cube and each directed to a tunable ring analysis cavity. For our working crystal temperature, wavelengths of signal and idler beams can differ by 0.8 to 0.9 nm. Analysis cavity bandwidths are $\delta\nu_{ac} = 14(1)$ MHz. The reflected field is detected by a high quantum efficiency [95(3)%] photodiode (Epitaxx ETX 300). The photocurrent is preamplified and the dc and high frequency (HF) components separated. The HF components are sent to a demodulating chain, where they are mixed with a sinusoidal reference at the analysis frequency $\Omega = 27$ MHz (with a filter bandwidth of 600 kHz). As the analysis cavities' resonances are swept over time, variances of each individual noise component, of their sum, and of their difference are calculated. The number N of points used, which is proportional to the acquisition time, is large enough to guarantee a well-defined variance and small enough to correspond to a very small change in cavities' detuning (this is equivalent to a low-pass filter of 1 kHz for the power spectrum).

Sum and difference noise spectra recorded as functions of the synchronous analysis cavity frequency scans are presented in Fig. 2. A scan over $\pm 3.2\delta\nu_{ac}$ is presented in Fig. 2(a). From the sum and difference spectra, one can easily notice that the quadrature being measured on each beam alternates between amplitude and phase. In particular, we recognize the amplitude quadrature behavior for detunings $|\Delta| \gtrsim 3\delta\nu_{ac}$ (carrier frequency and noise sidebands unaffected by cavity resonance) and for $\Delta = 0$ (carrier frequency gains a π phase shift with respect to sidebands). One observes that $\Delta^2 \hat{p}_-$ at $\Delta = 0$ does not recover the squeezing observed for $|\Delta| \gtrsim 3\delta\nu_{ac}$, owing to
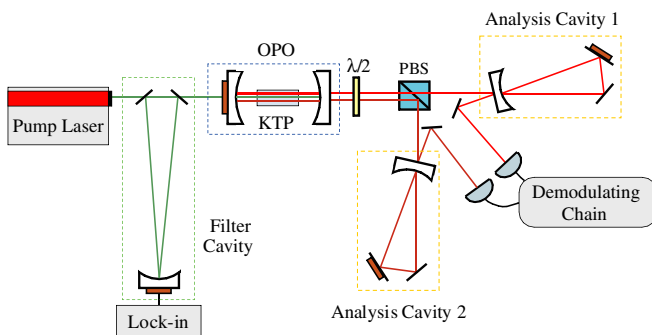
a lack of experimental resolution (few data points are obtained at any particular detuning). Phase quadrature is measured for $|\Delta| = 0.5\delta\nu_{ac}$ (carrier frequency gains a $\pi/2$ phase shift with respect to sidebands) and, for the particular choice of analysis frequency we use, also at $|\Delta| = 1.8\delta\nu_{ac}$ (one sideband gains a π phase shift relative to carrier frequency and to the other sideband). For all other detunings we measure a linear combination of amplitude and phase quadratures. In Fig. 2(b), we present a scan with increased resolution, over only $\pm 1\delta\nu_{ac}$. Phase-sum squeezing is observed, with $\Delta^2 \hat{q}_+ = 0.82(2)$. These spectra are individual scans and are representative of the best data obtained.

The solid curves in Figs. 2(a) and 2(b) are given by fits of Eq. (3) to the data. Apart from scale factors and curve central position, the relevant free parameters are the combined quadrature fluctuations $\Delta^2 \hat{p}_+$, $\Delta^2 \hat{p}_-$, $\Delta^2 \hat{q}_+$, and $\Delta^2 \hat{q}_-$. All other variables required in Eq. (3), such as
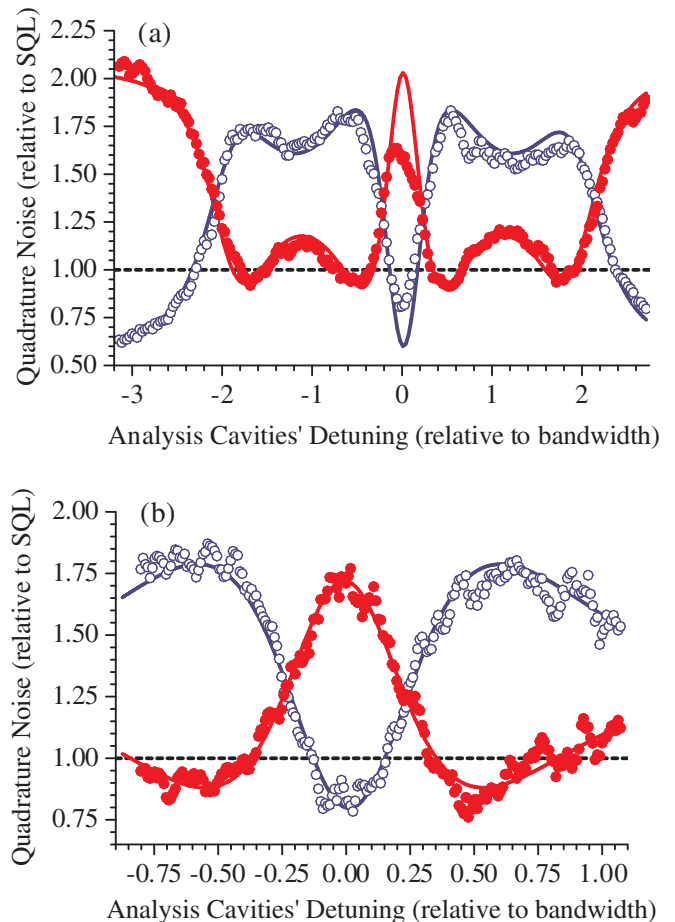

FIG. 2 (color online). Measurements of sum (full circles) and difference (open circles) of quadrature fluctuations as functions of analysis cavities' detunings. In (a) the detuning spans the range $\pm 3.2\delta\nu_{ac}$. In (b) an increased resolution scan is presented, in which we clearly observe $\Delta^2 \hat{p}_- < 1$ for $\Delta = 0$ and $\Delta^2 \hat{q}_+ < 1$ for $|\Delta| = 0.5\delta\nu_{ac}$, characterizing squeezed-state entanglement. The solid curves are given by fits of Eq. (3) to the data.

FIG. 1 (color online). Sketch of the experimental setup (details explained in the text).

cavity bandwidth and analysis frequency, are independently measured and employed as constants in the fitting, which is done either for the sum or for the difference in each part of Fig. 2, with equally good results.

The phase noise is strongly dependent on pump power relative to threshold. At 1.5 times the threshold power, we observed large excess noise in the phase sum, as was also found in [20]. This excess noise is not predicted by the standard OPO linearized theory [13], and it is still not clear whether a full quantum theory [24] can account for it. Although the calculations in [18] are carried out with the linearized theory, they provided us with a useful indication: squeezing in the phase sum improves as the system approaches the threshold. Consistently with this indication, we could only observe phase-sum squeezing for pump powers less than 7% above threshold. The best data shown in Fig. 2 were obtained approximately 4% above threshold. In this situation, the OPO is very unstable, hindering the measurements.

In terms of the entanglement criterion of Eq. (2), using $\Delta^2 \hat{q}_+ = 0.82(2)$, we obtain $\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.41(2) < 2$, a clear violation of the inequality, characterizing, for the first time, entanglement between bright beams of truly different frequencies. Equation (2) is a necessary and sufficient separability condition for Gaussian states, which are predicted for the OPO and are consistent with our data. Another, more stringent, entanglement criterion is the so-called EPR criterion, which enables one to infer variances on one beam, as functions of variances on the other beam [13]. This criterion implies a violation of the inequality $\Delta^2 \hat{p}_{\text{inf}} \Delta^2 \hat{q}_{\text{inf}} \geq 1$, where

$$\Delta^2 \hat{p}_{\text{inf}} = \Delta^2 \hat{p}_1 \left(1 - \frac{\langle \delta \hat{p}_1 \delta \hat{p}_2 \rangle^2}{\Delta^2 \hat{p}_1 \Delta^2 \hat{p}_2} \right), \tag{6}$$

with $\delta \hat{p}_i = \hat{p}_i - \langle \hat{p}_i \rangle$. An analogous relation holds for $\Delta^2 \hat{q}_{\text{inf}}$. Our measured data give the value $\Delta^2 \hat{p}_{\text{inf}} \Delta^2 \hat{q}_{\text{inf}} = 0.95(-3/+6)$, which is compatible with the violation of this second criterion as well. We have losses from the output of the OPO to the detectors. If the overall detection efficiency is η, the measured variances are related to the "true" variances (e.g., $\Delta^2 \hat{p}'$) by $\Delta^2 \hat{p} = \eta(\Delta^2 \hat{p}' - 1) + 1$. For $\eta = 80(2)\%$, we obtain $\Delta^2 \hat{p}'_- + \Delta^2 \hat{q}'_+ = 1.26(4)$ and $\Delta^2 \hat{p}'_{\text{inf}} \Delta^2 \hat{q}'_{\text{inf}} = 0.77(-9/+11)$.

The entanglement generated in this system can be substantially improved. The intensity difference squeezing measured in [20] reached the impressive value of 9.7 dB, which would lead to even stronger violations of the above criteria. Furthermore, stable operation only a few percent above threshold is technically feasible, as shown in [19]. Hence, the above-threshold OPO can produce very high degrees of entanglement, at least comparable to the best results obtained below threshold [7,9].

In summary, we have demonstrated, for the first time, bright two-color squeezed-state continuous variable entan-

glement. Applications to quantum information, such as quantum key distribution [25] and quantum teleportation [26] with continuous variables, can easily be envisaged. Our measurement scheme has interesting properties for both. Quantum key distribution with squeezed-state entanglement usually requires sending the local oscillator, in addition to the entangled field, so as to enable homodyne measurements, a requirement that is not necessary in our case. Teleportation, on the other hand, has been restricted to fields of the same frequency. Two-color entanglement opens the way for distributing quantum information between different parts of the electromagnetic spectrum.

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