Direct Production of Tripartite Pump-Signal-Idler Entanglement in the Above-Threshold Optical Parametric Oscillator

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We calculate the quantum correlations existing among the three output fields (pump, signal, and idler) of a triply resonant nondegenerate optical parametric oscillator operating above threshold. By applying the standard criteria [P. van Loock and A. Furusawa, Phys. Rev. A **67**, 052315 (2003)], we show that strong tripartite continuous-variable entanglement is present in this well-known and simple system. Furthermore, since the entanglement is generated directly from a nonlinear process, the three entangled fields can have very different frequencies, opening the way for multicolored quantum information networks.

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Entanglement, which is probably the strangest of all quantum phenomena, is considered the most important resource for future quantum information technology. Recent experiments on quantum computing, storage, and communication of information utilize different "quantum hardware," such as atom clouds [1], quantum dots [2], and trapped ions [3], all with different resonance frequencies. These systems will probably be used in nodes of quantum networks, implying the necessity of devising ways to address them without loss of quantum information. For networks with several nodes, multipartite entangled light beams will be important to carry out such tasks.

Most of the current realizations of entangled light beams are implemented by combining squeezed beams on beam splitters [4–9]. The beam splitter transformation is linear and does not lead to entangled beams of different frequencies. In order to produce multicolored entangled beams it is important to generate them directly from a nonlinear process. In the case of bipartite two-color entanglement, this has been done very recently, in the above-threshold optical parametric oscillator (OPO) [10–12].

The OPO is the best known and most widely used source of entangled continuous variables for quantum information purposes [13]. Nevertheless, focus thus far has been on the down-converted beams it produces, usually overlooking quantum properties of the pump beam. Recent proposals for direct generation of tripartite entanglement use so-called cascaded nonlinearities, combining down-conversion and sum or subtraction frequency generation [14], which are not present in standard OPOs. In this Letter, we theoretically demonstrate that the standard triply resonant above-threshold OPO naturally produces pump-signal-idler tripartite entanglement. We show that the down-converted and the pump fields' noises violate inequalities which are sufficient for witnessing entanglement [13]. We believe this to be the simplest and most practical proposal of a multicolored multipartite entanglement source to date.

For tripartite systems with subsystems (k, m, n), if the state is partially separable, the density operator can be written in the form of a statistical mixture of reduced density operators $\hat{\rho}_{i,km}$ and $\hat{\rho}_{i,n}$:

$$\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,km} \otimes \hat{\rho}_{i,n}, \tag{1}$$

with weights $\eta_i \geq 0$ satisfying $\sum_i \eta_i = 1$. A necessary condition for separability of two subsystems was demonstrated by Duan *et al.* [15], in the form of an inequality: if it is violated, there is bipartite entanglement. This criterion is easily checked experimentally by measuring second order moments of combinations of operators acting on each of the subsystems.

The inequality presented in Ref. [15] for the variances of two combinations of positions and momenta $(\hat{x}_j, \hat{p}_j)$ of subsystems $j = \{1, 2\}$ can be readily extended to a combination of three subsystems [16]. If we define two commuting operators $\hat{u} = h_1\hat{x}_1 + h_2\hat{x}_2 + h_3\hat{x}_3$ and $\hat{v} = g_1\hat{p}_1 + g_2\hat{p}_2 + g_3\hat{p}_3$, where the h_i and g_i are arbitrary real parameters, for a (partially) separable state written in the form of Eq. (1), inequalities of the form:

$$\langle \Delta^2 \hat{u} \rangle + \langle \Delta^2 \hat{v} \rangle \geq 2(|h_n g_n| + |h_k g_k + h_m g_m|), \tag{2}$$

with different permutations of the subsystems (k, m, n), must hold. Therefore, violations of the inequalities corresponding to the three possible permutations suffice to demonstrate tripartite entanglement.

For electromagnetic fields, position and momentum operators can be replaced by the field amplitude and phase quadrature operators, defined as functions of the creation and annihilation operators as $\hat{p}_j(t) = [e^{i\varphi_j}\hat{a}_j^\dagger(t) + e^{-i\varphi_j}\hat{a}_j(t)]$ and $\hat{q}_j(t) = i[e^{i\varphi_j}\hat{a}_j^\dagger(t) - e^{-i\varphi_j}\hat{a}_j(t)]$, where the phase φ_j of each mode is chosen from its mean value in order to have $\langle \hat{q}_j \rangle = 0$. In this case, $\hat{p}$ represents the amplitude fluctuations of the field, and $\hat{q}$ is related to the phase fluctuations. From the commutation relation

$[\hat{a}_j, \hat{a}^\dagger_{j'}] = \delta_{jj'}$, it follows that $[\hat{p}_j, \hat{q}_{j'}] = 2i\delta_{jj'}$. In the present situation, we look for violations of the following inequalities:

$$S_1 = \langle \Delta^2(\hat{p}_1 - \hat{p}_2)\rangle + \langle \Delta^2(\hat{q}_1 + \hat{q}_2 - \alpha_0 \hat{q}_0)\rangle \geq 4, \quad (3)$$

$$S_2 = \langle \Delta^2(\hat{p}_0 + \hat{p}_1)\rangle + \langle \Delta^2(\hat{q}_1 + \alpha_2 \hat{q}_2 - \hat{q}_0)\rangle \geq 4, \quad (4)$$

$$S_3 = \langle \Delta^2(\hat{p}_0 + \hat{p}_2)\rangle + \langle \Delta^2(\alpha_1 \hat{q}_1 + \hat{q}_2 - \hat{q}_0)\rangle \geq 4, \quad (5)$$

with an optimized choice of the free parameters α_i, in order to show that all three modes are entangled, i.e., that the state of the full system is not even partially separable.

The tripartite entangled fields are directly produced by a triply resonant nondegenerate OPO, composed of a $\chi^{(2)}$ nonlinear crystal placed inside an optical cavity (a full description of field mean values and tuning characteristics can be found in Ref. [17]). The OPO is a well-known source of nonclassical states of the electromagnetic field, both above and below the oscillation threshold. In this system, a pump photon of frequency ω_0 is down-converted into a pair of signal and idler twin photons of frequencies ω_1 and ω_2. These fields exit the cavity and can be easily separated by color (pump) and polarization (signal and idler) in the case of type-II phase matching. Below threshold, signal and idler modes are in an entangled state with zero mean values for the electric field [18]. Above threshold, the parametric coupling leads to both intensity coupling between the three modes (this is the well-known pump depletion effect: a pump photon is destroyed each time a couple of twin and idler photons is created) and to phase coherence between them: the sum of the signal and idler field phases is locked to the pump phase as a consequence of energy conservation ($\omega_1 + \omega_2 = \omega_0$). This leads to both intensity and phase correlations between the three modes that extend to the quantum regime, and eventually culminate in tripartite entanglement as we show below. So far, physicists' interest has been concentrated on the signal and idler quantum correlations [19] or on the pump squeezing [20]. The full three-mode system has indeed genuine quantum properties [21], which are partly lost when one traces out the pump mode, although the signal and idler modes remain of course entangled [10].

Quantum fluctuations of the system are calculated as usual [22]: we start from the evolution equations of the operators of the three modes ($\hat{a}_0$, $\hat{a}_1$, $\hat{a}_2$) inside the OPO cavity. We write the field operators as the sum of their mean values and a fluctuation term and, assuming that the fluctuations are small compared to the mean fields, which is true everywhere except very close to threshold, we linearize these equations around the classical mean values [17]. One obtains in this way six linear Langevin equations that enable us to calculate the evolution of the real and imaginary parts of the intracavity fluctuations of the three fields. If we assume that the cavity transmission factor and the extra-losses are the same for the signal and idler modes, the evolution equations can be decoupled into two independent sets [22]: two equations for the signal and idler difference, and four equations coupling the sum of the signal and idler fluctuations to the pump fluctuations. Using the input-output relation on the coupling mirror, one obtains the output field fluctuations in Fourier domain, $\delta\vec{p}(\Omega) = [\delta\hat{p}_0(\Omega), \delta\hat{q}_0(\Omega), \delta\hat{p}_1(\Omega), \delta\hat{q}_1(\Omega), \delta\hat{p}_2(\Omega), \delta\hat{q}_2(\Omega)]^T$, as a function of the input field fluctuations. This enables us to determine the full 6×6 three-mode covariance matrix, $C = \langle \delta\vec{p}(\Omega)\delta\vec{p}(-\Omega)^T\rangle$, of the pump, signal and idler output modes, and the variance of any combination of these modes. The full treatment is described in Ref. [23].

From the calculated covariances, we derive the optimized values of the parameters α_i which minimize the quantities S_1, S_2 and S_3 of Eqs. (3)–(5) as functions of the covariance matrix elements for the output field, and calculate the corresponding minimum value for these three quantities. We take typical experimental conditions: cavity coupling mirror transmittance for pump $T_0 = 10\%$ and signal and idler beams $T = 2\%$, and exact cavity resonance for the three modes. We can now study the dependence of S_1, S_2, and S_3 with the normalized pump power σ (power normalized to the oscillation threshold on resonance) and with the analysis frequency ω (normalized to the inverse of the cavity round trip time τ).

In Fig. 1, we display the minimized value of S_1. As can be seen, $S_1^{\min}$ is smaller than 4 in all the presented range of parameters, which establishes the inseparability of the signal and idler modes. Let us stress that the resulting violation, with the optimization of the variance, is much stronger than that observed by tracing out the pump mode and looking only at signal and idler modes under the Duan *et al.* criterion [23]. In the present case, the measurement of pump phase increases the knowledge that one can obtain about the idler beam phase from the measurement of the signal phase. Nevertheless, the state can still be partially separable if the other two inequalities [Eqs. (4) and (5)] are not violated. The interchangeability of the roles of signal and idler makes evident that $S_2 = S_3$. The common mini-

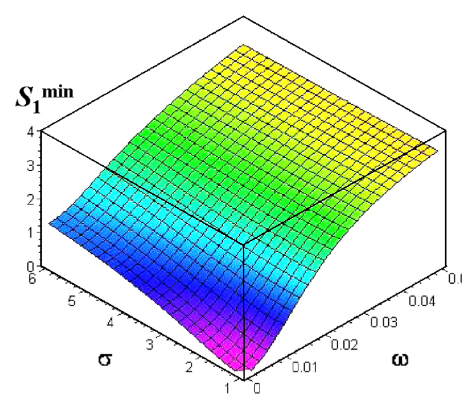

FIG. 1 (color online). Optimized sum of variances, $S_1^{\min}$, for Eq. (3): σ = pump power relative to threshold, ω = analysis frequency, in units of $1/\tau$.

mized value of this quantity is shown in Fig. 2. We observe that it is also below 4, implying inseparability for a broad range of values of analysis frequency and pump power, even though the violation is not as strong as for $S_1^{\min}$ (Fig. 1). Correlations between the twin beams tend to be stronger than those between one of the twins and the pump, since the pump is not generated inside the cavity. $S_2^{\min} = S_3^{\min}$ is everywhere larger than $\simeq 1.7$, a value obtained for $\sigma \simeq 1.6$. For this value of σ, all three fields have approximately the same intensities, which is in general the best situation for observing correlations.

Another method to characterize the amount of entanglement in a system is to study the eigenvalues of its covariance matrix: they provide information about the maximum squeezing that can be obtained from the different modes by unitary transformations and about the maximum bipartite entanglement that can be extracted from these modes [24]. In our case, the minimum eigenvalue is given by the variance of $\hat{p}_1 - \hat{p}_2$. The next lower eigenvalue is related to the combination of phases in the form $(\hat{q}_1 + \hat{q}_2 - \beta\hat{q}_0)$, where β is a real number. Both values can be strongly squeezed, at the expense of excess noise for the variances of their conjugate variables.

From these two smallest eigenvalues λ_1, λ_2 we calculate the logarithmic negativity $E_N = \max[0, -\log_2(\lambda_1\lambda_2)/2]$ [25,26]. This is a computable measure of the degree of bipartite entanglement of a system, and it is especially useful owing to its immediate extension to entangled mixed states. We calculate here the difference, E_N^{diff}, between the logarithmic negativities for the full system and for just the signal and idler modes, tracing out the pump. This difference is positive for the full range of parameters displayed in Fig. 3, with maximum values obtained for low analysis frequencies ($\omega < 0.02/\tau$). It is clear that quantum information is present in all three modes and one only recovers a fraction of it when restricting measurements to signal and idler beams.

The tripartite pump-signal-idler entanglement in the OPO can be observed in a broad range of frequencies and pump power. The correlation is, as expected, stronger for analysis frequencies below the cavity bandwidth T/τ for the signal and idler modes, and for pump powers close to threshold, although it does not depend so much on this last parameter. Calculations from the covariance matrix show that there is a small dependence of S_1 and S_2 on the cavity detunings, which is important because the locking of the OPO is typically done with some small detuning for pump and down-converted modes. If we consider the presence of spurious losses inside the cavity, there is a linear increase of the value of S_1 with these losses, much in the way observed for the intensity correlation of twin beams emitted from the OPO. As for S_2, inseparability no longer occurs for lower analysis frequencies, but still holds for a wide range of the parameters σ and ω.

In conclusion, we have demonstrated that the standard nondegenerate optical parametric oscillator directly yields tripartite entangled light beams when operating above threshold. Above-threshold OPOs have produced the highest level of intensity quantum correlations to date [27]. Figure 1 shows that they can also produce a very low bound for the combined phases quantum fluctuations. Thus, the magnitudes of expected quantum correlations are among the best achievable at present. The experimental realization of this system is much simpler than the proposals based on combined nonlinearities [28], especially considering the high degree of experimental control achieved over the OPO. We also note that the above-threshold OPO entanglement renders it a possible device for such tasks as a tripartite teleportation network [8]. Moreover, it allows distribution of quantum information through three modes of very different frequencies, a topic that is attracting growing attention [10,29,30]. This is of practical interest, since high efficiency photodetectors are only available in limited ranges of the electromagnetic spectrum. Frequency-tunable quantum information will also be very useful for light-matter interfaces in quantum networks.

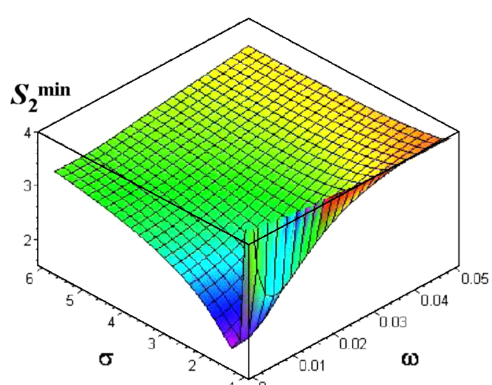

FIG. 2 (color online). Optimized sum of variances, $S_2^{\min} = S_3^{\min}$, for Eqs. (4) and (5): σ = pump power relative to threshold, ω = analysis frequency, in units of $1/\tau$.

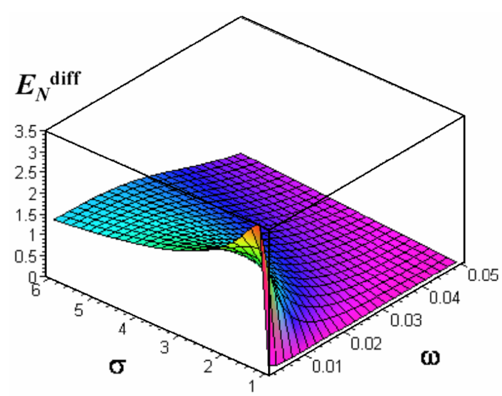

FIG. 3 (color online). Difference between logarithmic negativities, E_N^{diff}, for the full three modes and for only signal and idler modes. σ = pump power relative to threshold, ω = analysis frequency, in units of $1/\tau$.

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